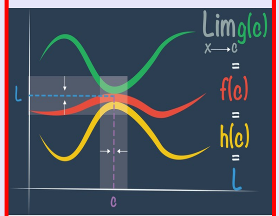
Math 261

Spring 2023

Lecture 9



Feb 19-8:47 AM

For $\varepsilon > 0$, find δ such that $\lim_{x \to 4} \left(\frac{1}{4}x - 1\right) = 0$ ✓

1) Verify the limit

$$\lim_{x \to 4} \left(\frac{1}{4}x - 1\right) = \frac{1}{4}(4) - 1 = 1 - 1 = 0 ✓$$

2) $f(x) = \frac{1}{4}x - 1$, $a = 4$, $L = 0$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{4}x - 1 - 0\right| < \varepsilon$ $\qquad$ $|x - 4| < \delta$

$\left|\frac{1}{4}x - 1\right| < \varepsilon$

$\left|\frac{1}{4}(x - 4)\right| < \varepsilon$

$\left|\frac{1}{4}\right| |x - 4| < \varepsilon$

$\frac{1}{4}|x - 4| < \varepsilon$ → Multiply by 4

$|x - 4| < 4\varepsilon$

So we pick

$\boxed{\delta = 4\varepsilon}$

Feb 21-8:47 AM

For any $\varepsilon > 0$, find δ such that $\lim_{x \to -5} \left(-\frac{2}{5}x + 4\right) = 6$.

1) $\lim_{x \to -5} \left(-\frac{2}{5}x + 4\right) = -\frac{2}{5}(-5) + 4 = 2 + 4 = 6$ ✓

2) $f(x) = -\frac{2}{5}x + 4$, $a = -5$, $L = 6$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|-\frac{2}{5}x + 4 - 6\right| < \varepsilon$ " $|x - (-5)| < \delta$

$\left|-\frac{2}{5}x - 2\right| < \varepsilon$ " $|x + 5| < \delta$

$\left|-\frac{2}{5}(x + 5)\right| < \varepsilon$

$\left|-\frac{2}{5}\right| |x + 5| < \varepsilon$

$\frac{2}{5}|x + 5| < \varepsilon$

→ Multiply by $\frac{5}{2}$

$|x + 5| < \frac{5}{2}\varepsilon$

$\boxed{\text{Pick } \delta = \frac{5\varepsilon}{2}}$

Feb 21-8:53 AM

For any $\varepsilon > 0$, find δ such that $\lim_{x \to 2} (x^2 + 5x) = 14$ ✓

1) Verify the limit

$\lim_{x \to 2} (x^2 + 5x) = 2^2 + 5(2) = 14$ ✓

2) $f(x) = x^2 + 5x$, $a = 2$, $L = 14$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 + 5x - 14| < \varepsilon$ " $|x - 2| < \delta$

$|(x + 7)(x - 2)| < \varepsilon$

$|x + 7|\,|x - 2| < \varepsilon$

Bound Keep

If $|x + 7| < C$, then $|x + 7||x - 2| < C|x - 2| < \varepsilon$

$|x - 2| < \frac{\varepsilon}{C}$

Pick $\delta = \frac{\varepsilon}{C}$

If we wish δ to be no more than 1 ⇒

$|x - 2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

$1 + 7 < x + 7 < 3 + 7$

$-10 < 8 < x + 7 < 10$

$-10 < x + 7 < 10$

$|x + 7| < 10$

$C = 10$

$\delta = \min\left\{1, \frac{\varepsilon}{10}\right\}$

Feb 21-9:03 AM

For $\varepsilon > 0$, find $0 < \delta \leq 1$ such that

$\lim_{x \to 1} (2x^2 - 5x) = -3.$

1) Verify limit ✓ 2) $f(x) = 2x^2 - 5x$, $a = 1$, $L = -3$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|2x^2 - 5x - (-3)| < \varepsilon$ " $|x - 1| < \delta$

$|2x^2 - 5x + 3| < \varepsilon$ " $|x - 1| < \delta$

$|(2x - 3)(x - 1)| < \varepsilon$ $\delta = \frac{\varepsilon}{C}$

Bound Keep

$|2x - 3|\,|x - 1| < \varepsilon$

If $|2x - 3| < C$, then $|x - 1| < \frac{\varepsilon}{C}$

If we wish δ to be no more than 1 $\Rightarrow$ $|x - 1| < 1$

$\delta \leq 1$

$-1 < x - 1 < 1$

$0 < x < 2$

$0 < 2x < 4$

$0 - 3 < 2x - 3 < 4 - 3$

$-3 < 2x - 3 < 1 < 3$

$-3 < 2x - 3 < 3$

$|2x - 3| < 3$

C

$\delta = \min\left\{1, \frac{\varepsilon}{3}\right\}$

If $\varepsilon = 1 \to \delta = \frac{1}{3}$

If $\varepsilon = 2 \to \delta = \frac{2}{3}$

If $\varepsilon = 3 \to \delta = 1$

If $\varepsilon = 6 \to \delta = \min\left\{1, \cancel{\frac{6}{3}}^{\,2}\right\} = 1$

Feb 21-9:12 AM

For $\varepsilon > 0$, find δ such that

$\lim_{x \to 1/2} \frac{1}{x} = 2$ ✓

1) $f(x) = \frac{1}{x}$, $a = \frac{1}{2}$, $L = 2$

2) $\lim_{x \to 1/2} \frac{1}{x} = \frac{1}{1/2} = 1 \div \frac{1}{2} = 1 \cdot \frac{2}{1} = 2$ ✓

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

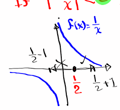
$\left|\frac{1}{x} - 2\right| < \varepsilon$ " $|x - \frac{1}{2}| < \delta$

$\left|\frac{-2}{x}\left(x - \frac{1}{2}\right)\right| < \varepsilon$ " $|x - \frac{1}{2}| < \delta$

$\left|\frac{-2}{x}\right|\left|x - \frac{1}{2}\right| < \varepsilon$ Pick $\delta = \frac{\varepsilon}{7}$

$\left|\frac{2}{x}\right|\left|x - \frac{1}{2}\right| < \varepsilon$

If $\left|\frac{2}{x}\right| < C$, then $\left|x - \frac{1}{2}\right| < \frac{\varepsilon}{C}$



Can $\delta = 1$? NO $f(x)$ Not defined at 0.

Can $\delta = \frac{1}{2}$? NO Same reason

Pick $\delta \leq \frac{1}{4}$ $\Rightarrow$ $\left|x - \frac{1}{2}\right| < \frac{1}{4}$

$-\frac{1}{4} < x - \frac{1}{2} < \frac{1}{4}$

$-\frac{1}{4} + \frac{1}{2} < x < \frac{1}{4} + \frac{1}{2}$

$\frac{1}{4} < x < \frac{3}{4}$

Make reciprocal

$4 > \frac{1}{x} > \frac{4}{3}$

$\frac{4}{3} < \frac{1}{x} < 4$

Multiply by 2

$\frac{8}{3} < \frac{2}{x} < 8$

$-8 < \frac{8}{3} < \frac{2}{x} < 8$

$-8 < \frac{2}{x} < 8$

$\left|\frac{2}{x}\right| < 8$

Pick $\delta = \min\left\{\frac{1}{4}, \frac{\varepsilon}{8}\right\}$

If $\varepsilon = 1 \to \delta = \frac{1}{8}$

If $\varepsilon = 3 \to \delta = \min\left\{\frac{1}{4}, \frac{3}{8}\right\}$

$= \min\{.25, .375\} = .25$

Feb 21-9:25 AM

For $\varepsilon > 0$, find $\delta > 0$ such that

$$\lim_{x \to 3} \frac{3}{x} = 1$$

1) limit verified ✓

2) $f(x) = \frac{3}{x}$, $a = 3$, $L = 1$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left| \frac{3}{x} - 1 \right| < \varepsilon$ " $|x-3| < \delta$

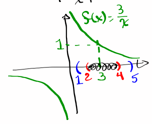
$\left| \frac{-1}{x}(x-3) \right| < \varepsilon$ " $|x-3| < \delta$

$\left| \frac{-1}{x} \right| |x-3| < \varepsilon$

$\underbrace{\left| \frac{1}{x} \right|}_{\text{Bound}} \underbrace{|x-3|}_{\text{Keep}} < \varepsilon$

$\delta = \frac{\varepsilon}{C}$

If $\left| \frac{1}{x} \right| < C$, then $|x-3| < \frac{\varepsilon}{C}$



we wish $\boxed{\delta \text{ to be no more than } 1}$

$|x-3| < 1$

$-1 < x-3 < 1$

$2 < x < 4$

$\frac{1}{2} > \frac{1}{x} > \frac{1}{4} \Rightarrow \frac{1}{4} < \frac{1}{x} < \frac{1}{2}$

$-\frac{1}{2} < \frac{1}{x} < \frac{1}{2}$

$\left| \frac{1}{x} \right| < \frac{1}{2}$

Pick

$\delta = \min\left\{ 1, \frac{\varepsilon}{\frac{1}{2}} \right\}$

$\delta = \min\{1, 2\varepsilon\}$

If $\varepsilon = .1 \rightarrow \delta = \min\{1, .2\} = .2$

If $\varepsilon = .4 \rightarrow \delta = \min\{1, .8\} = .8$

If $\varepsilon = .6 \rightarrow \delta = \min\{1, 1.2\} = 1$

Feb 21-9:43 AM